MPZ3132-Engineering Mathematics IB

Academic Year 2014/2015

Assignment NO.01

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1. (a) Find the period of the following functions:

   i. $\cos x$

   ii. $\sin 4x$

   iii. $\sin 3x$

   (b) Let$f$: $\mathbb{R} \to \mathbb{R}$ be a function defined by,

   $$f(x) = \begin{cases} 0, & \text{if } -\pi \leq x \leq 0; \\ 1, & \text{if } 0 < x \leq \pi. \end{cases}$$

   and $f(x + 2\pi) = f(x)$. Find the Fourier series expansion for $f(x)$.

   Hence, deduce that $f(x) = \dfrac{1}{2} + \dfrac{2}{\pi}\displaystyle\sum_{k=0}^{\infty} \frac{\sin(2k+1)x}{(2k+1)}$.

   (c) Expand the function $f(x) = e^{-t/2}$ in a Fourier series for the interval $0 \leq t \leq \pi$.

   (d) Find the Fourier sine expansion of,

   $$f(x) = \begin{cases} x, & \text{if } 0 < x \leq 1; \\ 2 - x, & \text{if } 1 < x < 2. \end{cases}$$

   and hence find the sub series of $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$

2. (a) Find the Fourier series of following function $f(x) = \begin{cases} 1, & \text{if } 0 < x < \pi; \\ -1, & \text{if } -\pi < x < 0. \end{cases}$

   Using the Parseval's formula deduce that

   $$\sum_{n=0}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

   (b) Let f be a periodic function of period 2π such that $f(x) = \pi^2 - x^2$ for $x \epsilon (-\pi, \pi)$

   Find the Fourier series expansion for $f(x)$.

(c) Expand $f(x) = |x|$ in a Fourier series for $-\pi < x < \pi$ and deduce that

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$$

3. Let $f(x)$ be a function of period 2π such that. $f(x) = \begin{cases} x, & 0 < x < \pi; \\ \Pi, & \pi < x < 2\pi. \end{cases}$

   (a) Sketch a graph of $f(x)$ in the interval $-2\pi < x < 2\pi$.

   (b) Show that the Fourier series of $f(x)$ in the interval $0 < x < 2\pi$ is
   $\frac{3\pi}{4} - \frac{2}{\pi}[\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + ...] - [\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + ...]$.

   (c) By giving appropriate value to x, show that

      i. $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$

      ii. $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + ...$

4. (a) Define the n$^{\text{th}}$ Taylor polynomial of $f(x)$ about $x = a$.
   If $f(x) = \dfrac{1 - x^2}{(1 - 2x\cos\theta + x^2)}$, then show that
   $(1 + x^2 - 2x\cos\theta)\dfrac{df^5(x)}{dx^5} + 10(x - \cos\theta)\dfrac{df^4(x)}{dx^4} + 20\dfrac{df^3(x)}{dx^3} = 0$.
   Deduce that the Taylor polynomial of $f(x)$ about $x = 0$ is $1 + \sum_{r=1}^{5} 2\cos(r\theta)x^r$.

   (b) Expand $f(x) = \dfrac{1}{1-x} - 1$ around $x = 0$, to get linear, quadratic and cubic approximation.

   (c) Find the 5^{th} Taylor Polynomial of the following functions around $x = 0$.

      i. $f_1(x) = \ln(2 - x)$

      ii. $f_2(x) = \dfrac{1}{(1-x)^2}$

5. (a) Find the Taylor polynomial of following functions:

   i. $\ln(\cos x)$

   ii. e^x

   Hence evaluate the following limits. $\lim_{x\to 0} \frac{\ln(\cos x)}{x^2}$ and $\lim_{x\to 0} \frac{x^2 e^x}{(\cos x - 1)}$

   (b) Prove that the Taylor series expansion about $x = 0$ of $\sin x$ and $\cos x$ are $\sum_{r=0}^{\infty} \frac{(-1)^r (x)^{2r+1}}{(2r+1)!}$ and $\sum_{r=0}^{\infty} \frac{(-1)^r x^{2r}}{(2r)!}$ respectively.
   Hence find the Taylor series expansion of $\sin^2 x$ and $\cos^2 x$ about x=0.